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On the Theory of Astronomical Refractions.

By Sir John W. Lubbock, Bart. F.R.S.*

In the year 1840 the author of this paper investigated the subject of Astronomical Refractions upon a different hypothesis of the constitution of the atmosphere from that assumed by preceding inquirers, and instituted a comparison between the results and the corresponding refractions inserted in the *Connaissance des Temps*, as well as those of Ivory and Bessel. The present communication, besides embodying the substance of his researches on that occasion, contains also some additional remarks and comparisons tending to elucidate more fully his previous labours.

The following table exhibits the constitution of the atmosphere, which the author was induced to adopt as the basis of his investigation:—

Height in Miles.	Pressure p. Inches.	Temperature τ. Fahr.	Density ϱ.
0	30·00	+ 50·0	1·00000
1	24·61	35·0	·84611
2	20·07	19·5	·71294
3	16·25	+ 3·4	·59798
4	13·06	− 13·3	·49903
5	10·41	30·6	·41403
10	2·81	126·4	·14499
15	·45	240·6	·03573
22·35	...	−448·0	...

We extract the following remarks:—

"Ivory, in his paper on refractions published in the *Philosophical Transactions* for 1838, instituted a comparison between the result of his theory and Bessel's table; and in p. 224 of that paper a table is given offering a comparison between the table of the *Conn. des Temps*, his own, and Bessel's. When afterwards, in the year 1840, I endeavoured to give a solution of this problem, assuming an atmosphere differently constituted from that of Ivory, and, in my opinion, nearer the truth, I also published the table, in p. 133, giving a similar comparison, and I took for Bessel's refractions those given as such by Ivory. It was not until very recently that I perceived that Ivory had altered all Bessel's figures by adding to the logarithm of the refraction according to Bessel, a constant quantity. As Ivory's description of the nature of this alteration appears to me to be somewhat obscure, I applied to Mr. Adams for his opinion, and he writes to me thus: 'Ivory's object, in his comparative table of refractions, is to compare the *laws of variation* of the refraction corresponding to different theories as to the constitution of the atmosphere, and not the *absolute amount* of the refraction, which will, of course, depend upon the assumed refractive power of air of a given density. For

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this purpose he increases the logarithms of the refractions given in Bessel's table, by the constant ·00507, in order to make the refraction at Z.D. 45° agree with his own value. According to Bessel himself the logarithm to be added to the quantity in table 1, in order to reduce the state of barometer and thermometer supposed by Ivory, would be ·00412 or ·00095 less than before; but this only indicates that Bessel supposes the refractive power of air, under given circumstances, to be slightly less than Ivory takes it. The latter assumes this refractive power to be the same as that on which the French tables are based, and consequently the quantities in these tables are rendered comparable with his own by merely adding to these logs. the constant ·00115, which is the difference of the log. of 29·921 inch. and 30 inch. in the heights of the barometer supposed in the two cases respectively.'

"Ivory regarded, and I think truly, that Bessel's refractions were to be considered as resulting from, and agreeing with, observation, and therefore as affording the best test which he could obtain of the accuracy of his own views touching the constitution of the atmosphere. That they are employed in the reduction of the *Greenwich Observations* shows the value which is attached to them by the Astronomer Royal.

"The value of the refraction at 45° apparent zenith distance of Bessel, as given by Ivory, is $58''{\cdot}36$, but the true value is $58''{\cdot}23$; this difference is so small that without recalculating my expressions, which would involve a very serious amount of labour, the proper figures may be obtained, owing to the manner in which the constant depending on the refractive power of air enters into this expression by subtracting from the refraction, as given in my former table, a small quantity proportioned to it. In this manner the first column of the following table has been formed. The second column has been calculated by me from the tables contained in the *Greenwich Observations* for 1853, recently issued, the argument being the apparent zenith distance for barometer 30 inch. and 50° Fahrenheit. The third column was calculated by me from the tables formed upon Laplace's theory, and given in the *Conn. des Temps* for 1851, by M. Caillet. At low altitudes these tables give greater refractions than Bessel's, but the table of errors which Bessel gives in the *Fundamenta*, p. 53, seems to indicate that his refractions are too great near the horizon. The fourth column gives the refractions, which would obtain, calculated from Laplace's expressions, if the refraction at 45° was made to agree with Bessel's, that is, they are the numbers in col. 3 multiplied by $\frac{58{\cdot}23}{58{\cdot}39}$. But it must be recollected that astronomers who reduce their observations by means of M. Caillet's tables use those given in col. 3.

"My expression for the refraction, barometer 30 inch. and Fahrenheit 50°, is as follows, the coefficients of my former expression being multiplied by $\frac{58{\cdot}23}{58{\cdot}36}$:

$$\text{Ref.} = \sin\theta\,\{1130''{\cdot}3\,e + 636''{\cdot}8\,e^3 + 219''{\cdot}9\,e^5 + 60''{\cdot}4\,e^7 + 17''{\cdot}8\,e^9 + 5''{\cdot}5\,e^{11} + \&c.\}$$

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"When θ is the apparent zenith distance, $e = \tan \frac{\phi}{2}$

$$\tan \phi = \frac{[9\cdot0139814]}{\cos \theta}$$

"This expression gives for the horizontal refraction 2070″·7. Ivory says that there is great probability that the horizontal refraction is very near 2070″, and does not exceed that quantity.

"Groombridge, who made many observations for the purpose of determining the amount of the refraction near the horizon, makes the horizontal refraction for barometer 30 inch., and thermometer Fahrenheit 50°, 2075″·4, which is exactly the horizontal refraction furnished in my expression in p. 134.

"It may be concluded that the refractions which belong to the atmosphere, constituted as I have supposed, in conformity with my theory of the heat of steam and other vapours, are consistent with observation.

Mean Refractions for the Temperature 50 *Fahrenheit and Barometric Pressure* 30 *Inches.*

App. Zen. Dist.	Lubbock. 1855.	Bessel.	Conn. des Temps.	Conn. des Temps, alt^{d}.	App. Zen. Dist.
10°	10″·28	10″·28	10″·31	10″·28	10°
20	21·21	21·21	21·27	21·21	20
30	33·65	34·65	33·74	33·65	30
40	48·88	48·88	49·01	48·88	40
45	58·23	58·23	58·39	58·23	45
50	69·36	69·36	69·56	69·37	50
55	83·06	83·06	83·29	83·06	55
60	100·62	100·62	100·77	100·49	60
65	124·35	124·34	124·70	124·36	65
70	158·81	158·76	159·23	158·79	70
75	214·20	214·10	214·78	214·19	75
80	319·37	319·16	320·23	319·35	80
85	591·81	591·99	591·70	590·07	85
86	704·46	705·40			86
87	862·27	862·49			87
88	1094·76	1098·70			88

"As Table V. is not carried beyond 85° in the *Conn. des Temps*, I calculated from Bessel's tables the refractions for 86°, 87°, and 88°, for Fahrenheit 50°, and barometer 29·921, in order to compare them with those given by M. Caillet for those conditions in Table VI., *Conn. des Temps*, 1851, p. 58.

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	Bessel.	Conn. des Temps.	Conn. des Temps, altd.
86	703·63	708·80	706·85
87	860·26	868·73	866·35
88	1095·80	1103·09	1100·07

Dr. Drew exhibited a series of astronomical diagrams, representing various celestial objects, such as the phases of the planets, the craters in the moon, clusters of stars, nebulæ, &c., executed from the drawings of Mr. De La Rue, Lord Rosse, Sir John Herschel, and other original sources, and adapted to the lecture-room. The nebulæ and double-stars were depicted on a blue ground, which produced a pleasing effect.

Note on Comet II. 1855. By Dr. Donati.

In this Note Dr. Donati first gives parabolic elements of Comet II. 1855, and then assigns elliptic elements which he deduced from four positions of the comet by the aid of a method recently devised by Mossotti. The following are the positions which served as the basis of calculation; they are reduced to the mean equinox of Jan. 1, 1855, and corrected for aberration and parallax:—

Florence M.T.	R.A. ° ′ ″	Decl. ° ′ ″
1855, Jan. 3·41956	100 17 2·8	+36 19 42·0
5·44491	107 49 57·1	36 15 48·9
11·42931	121 9 45·1	34 39 36·1
17·41726	127 23 38·8	+33 2 16·1

From these positions the following elliptical elements were deduced:—

Passage of the Perihelion, 1855, May 30·232563 Florence M.T.

	° ′ ″	
Longitude of the Ascending Node......	260 15 7·3	Mean Equinox, 1855·0.
Longitude of the Perihelion	282 54 12·7	
Inclination	156 52 51·6	
Perihelion Distance......................	0·5678239	
Excentricity..............................	0·9909006	

whence

Semi-major Axis	62·40234
Time of Revolution......	492·95 years.